SF-6573/300

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B. E. (II) (Sem. - IV) (Mech.) Examination

May/June – 2011

Engineering Mathematics - III

Time : 3 Hours] [Total Marks : 100

Instruction :

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. (2) (SEM. - 4) (MECH.)**

Name of the Subject :
☛ **ENGINEERING MATHEMATICS - 3**

☛ Subject Code No. : **6 5 7 3** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

1 a) Attempt the following. [10]

1) Evaluate $\int_0^1\int_0^x e^{y/x}\,dydx$.

2) Express the following integral as equivalent integral by using cylindrical co-ordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} dzdydx$.

3) Show that $grad\left[f(r)\right] = f'(r)\cdot\frac{\vec{r}}{r}$.

4) Show that $div(curl\vec{F}) = 0$

5) If $f(x)$ is periodic function of x of fundamental period p, then show that $f(ax), a \neq 0$ is a periodic function of x of period $p/_a$.

b) Attempt any two of the following. [10]

1) Evaluate $\iint_R xy\cdot dA$ over the region R enclosed between $y=\frac{x}{2}, y=\sqrt{x},$ x=2 & x=4.

2) Evaluate $\iint_R \sqrt{xy-y^2}\,dA$, Where R is triangle with vertices (0,0),(10,1) & (1,1).

3) Find the volume common to the cylinder $x^2+y^2=a^2$ and $x^2+z^2=a^2$.

2 a) Verify divergence theorem for $\vec{F} = 4xi - 2y^2j + z^2k$ taken over the region bounded by the cylinder $x^2+y^2=4, z=0, z=3$. **[6]**

b) Attempt any three of the following. [9]

1) A vector field is given by $\vec{F} = (x^2+xy^2)i + (y^2+x^2y)j$. Show that $\vec{F}$ is irrotational and find its scalar potential.

2) A vector field is given by $\vec{F} = (\sin y)i + x(1+\cos y)j$. Evaluate the line integral $\oint_C \vec{F}\cdot dr$ over the circular path $x^2+y^2=a^2, z=0$.

3) Evaluate $\iint_S \vec{r}\cdot\hat{n}\, dS$, where S is a closed surface.

4) Show that for any closed surface σ, $\iint_\sigma Curl\vec{F}\cdot\hat{n}\, d\sigma = 0$

3 a) Define Fourier series and obtain Euler's formula for Fourier coefficient for the periodic function in the interval $0 < x < 2l$. **[5]**

b) **Attempt any two of the following.** **[10]**

1) Obtain the Fourier series to represent $f(x) = \frac{1}{4}(\pi - x)^2, 0 < x < 2\pi$.

2) Find the Fourier series to represent the function $f(x)$ given by

$$f(x) = \begin{cases} x & for\ 0 \le x \le \pi \\ 2\pi - x & for\ \pi \le x \le 2\pi \end{cases}$$. Deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

3) Find half-range cosine series for the function $f(x) = x^2, 0 \le x \le \pi$.

4 a) **Attempt the following.** **[10]**

1) Show that error function is an odd function.

2) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\cdot d\theta$.

3) Solve $2p + 3q = 1$.

4) Define the Lagrange linear differential equation & state the algorithm to find the two linearly independent solutions of it.

5) During war one ship out of nine was sunk on an average in making a voyage. Find the probability that exactly 2 out of a convoy of five ships would arrive safely?

b) State & prove relation between $Beta\ \&\ Gamma\ function.$ **[04]**

c) **Attempt any two of the following.** **[06]**

1) Prove that $\int_0^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$.

2) Prove that $\int_0^1 \left(1 - \sqrt[n]{x}\right)^m dx = \frac{m!n!}{(m+n)!}$.

3) Prove that $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{1}{a^n b^m}\beta(m,n)$.

5 a) Solve the following One-dimensional wave equation using separation of variables method. **[08]**

$u_{tt} = c^2 u_{xx}$

$u(0,t) = u(l,t) = 0;\ for\ t > 0$

$u(x,0) = f(x)\ ;for\ 0 < x < l$

$u_t(x,0) = g(x)\ ;for\ 0 < x < l$

b) **Attempt any one of the following.** **[07]**

1) An insulated metal bar, 10 units long has the temperature bar of its ends maintained at 0^0C and t=0 the temperature distribution along the bar is defined by $f(x) = x(10 - x)$. Determine temperature $u(x,t)$ at any point in the bar at time t. $[Take\ c^2 = 4]$

2) A rectangular plate with insulated surfaces is 10 cm. Wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. If the temperature along short edge y=0 is given by $u(x,0) = \begin{cases} 20x, & 0 < x \leq 5 \\ 20(10-x), & 5 \leq x < 10 \end{cases}$;while the other two edges x=0 and x=10 as well as the other short edges are kept at 0^0C. find the steady state temperature at any point of the plate.

6 a) **Attempt any two of the following.** **[06]**

1) $(x^2 - yz)p + (y^2 - xz)p = z^2 - xy$.

2) $z(xp - yq) = y^2 - x^2$

3) $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$

b) **Attempt any three of the following.** **[09]**

1) Assume that the probability of an individual coalminer being killed in a mine accident during a year is $\frac{1}{2400}$. Use Poisson distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year.

2) In a normal distribution 31% of items are under 45 and 8% are above 64. Find the mean and standard deviation of the distribution.
$[S.N.V(0.5) = 0.19\ area\ and\ S.N.V(1.4) = 0.42\ area]$

3) The average income of persons was Rs. 210/- with standard deviation of Rs. 10/- in sample of 100 people of a city. For another sample of 150 persons, the average income was Rs. 220/- with standard deviation of Rs. 12/-. The standard deviation of all people of the city was Rs. 11/-. Test whether there is any significant difference between the average incomes of the localities.
$[The\ value\ of\ z\ at\ 5\%\ level\ of\ signifinance\ is\ 1.96.]$

4) A sample of 20 items has mean 42 units and standard deviation 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.
$[t\ at\ 5\%\ level\ of\ significance\ for\ 19\ degree\ of\ freedom\ is\ 2.09]$